Eulerizing and Semi-Eulerizing Graphs

Lecture 28
Section 5.4

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Wed, Nov 7, 2018

Outline

Definitions

Definition (Eulerization)

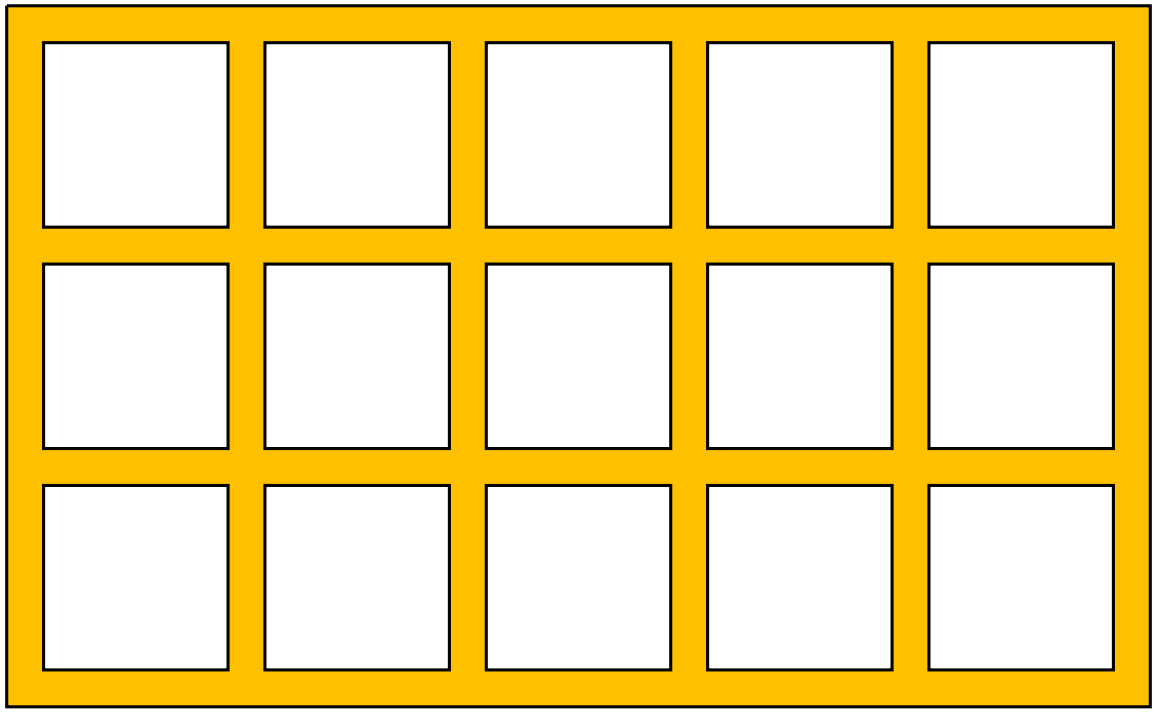
To eulerize a graph is to add *exactly* enough edges so that every vertex is even. Then an Euler circuit will exist.

Definition (Semi-Eulerization)

To semi-eulerize a graph is to add *exactly* enough edges so that all but two vertices are even. Then an Euler path will exist.

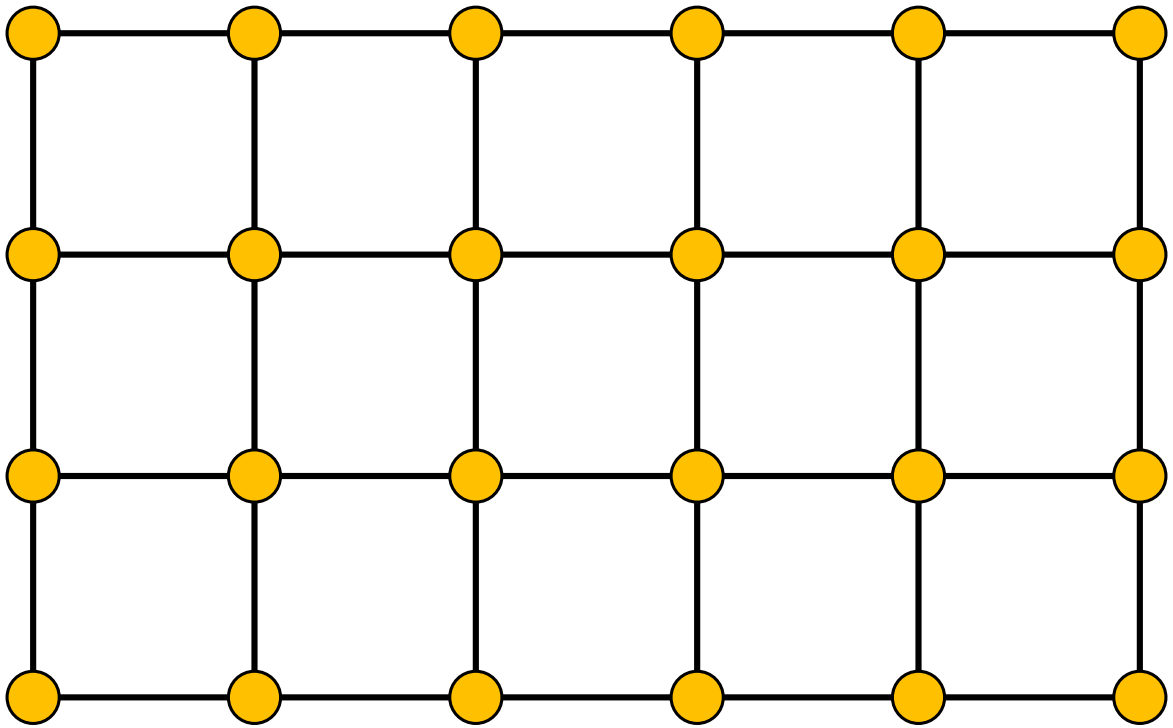
Outline

The Security Guard Problem



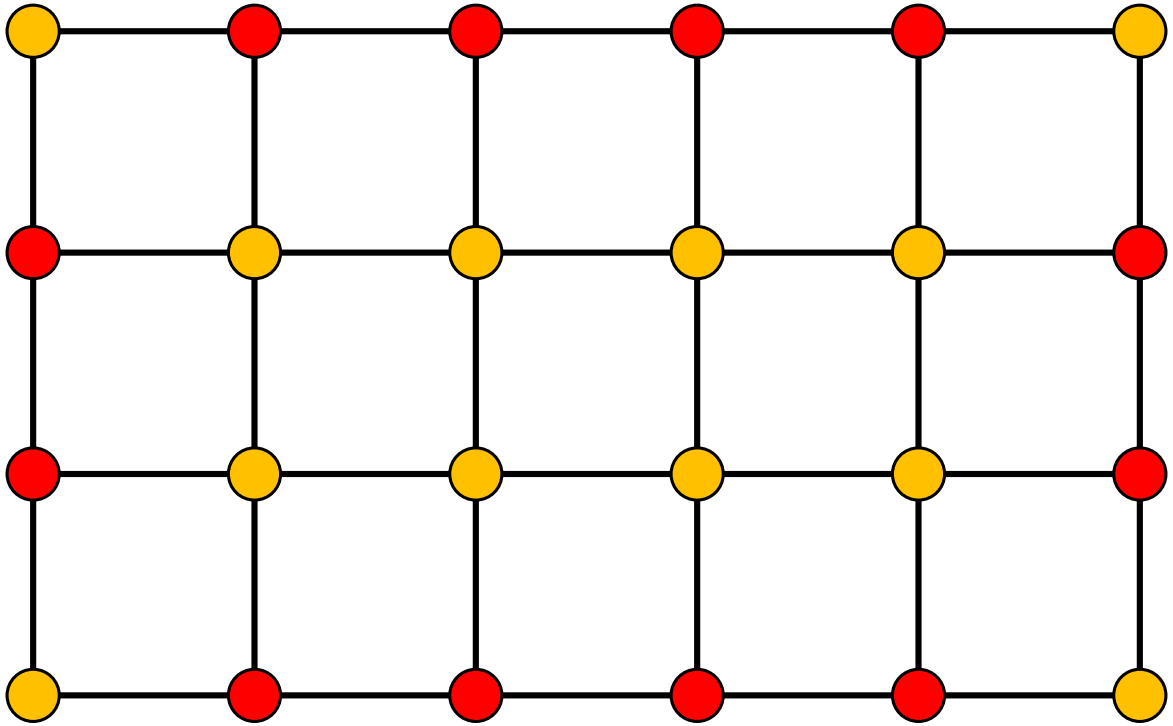
The neighborhood.

The Security Guard Problem



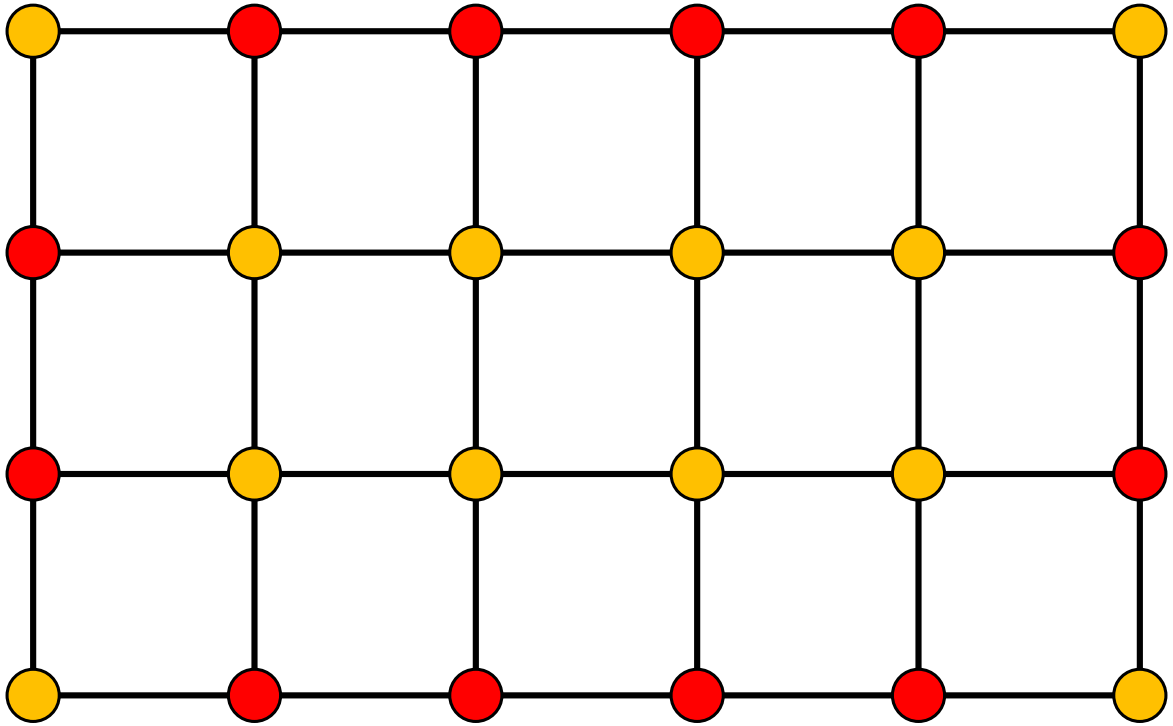
The neighborhood as a graph.

The Security Guard Problem



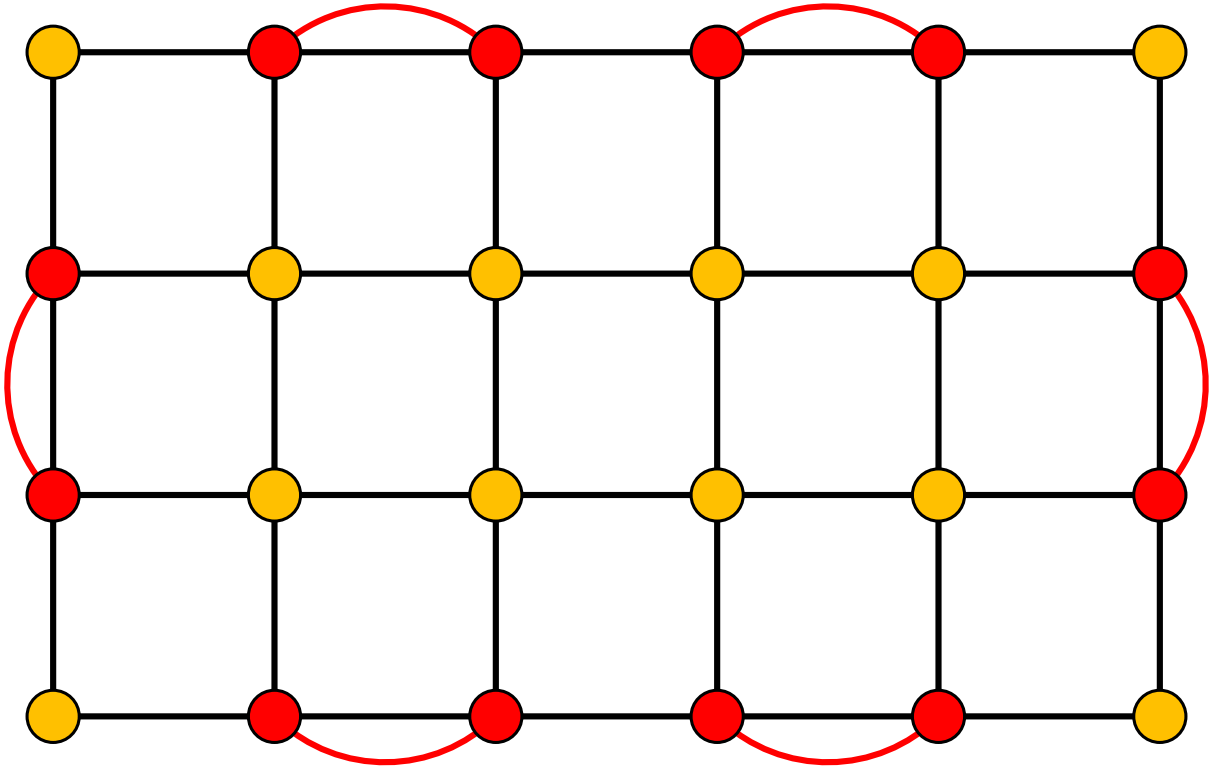
There are 12 odd vertices.

The Security Guard Problem



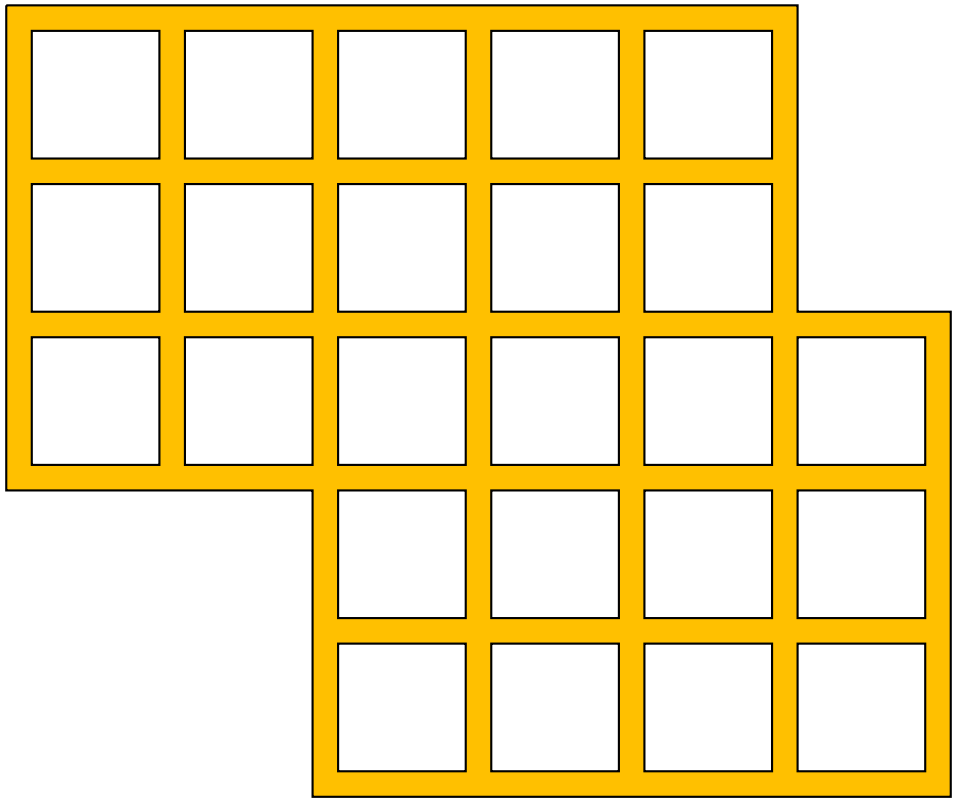
At least 6 edges must be added. Why 6?

The Security Guard Problem



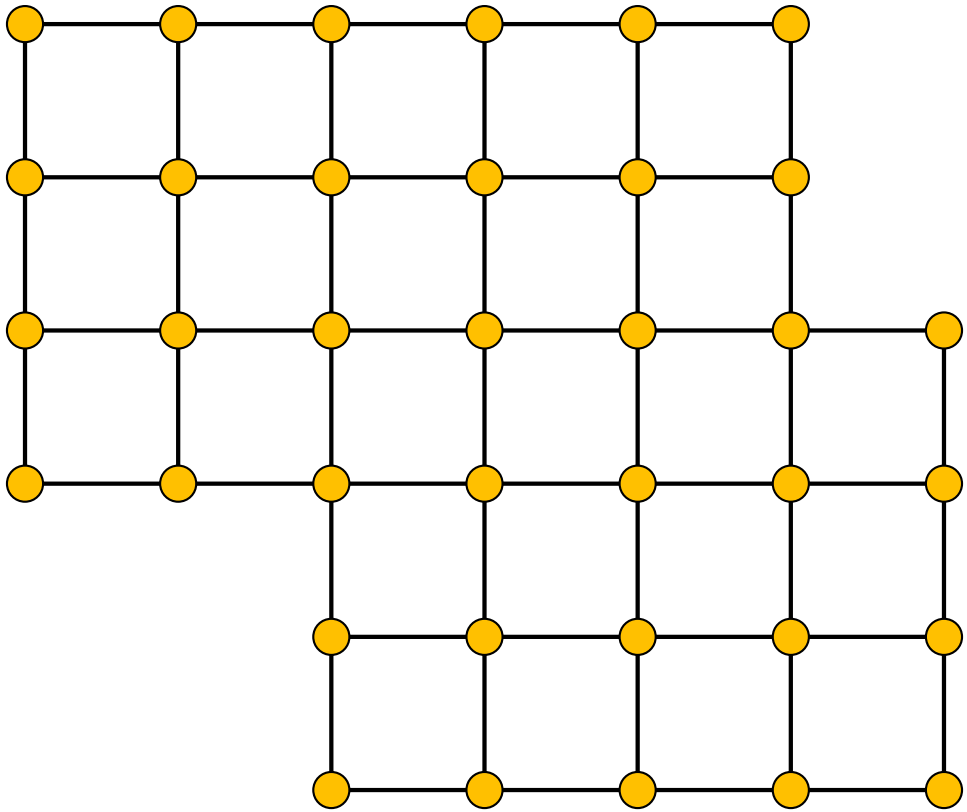
This solution uses 6 new edges. Is it optimal?

The Security Guard Problem



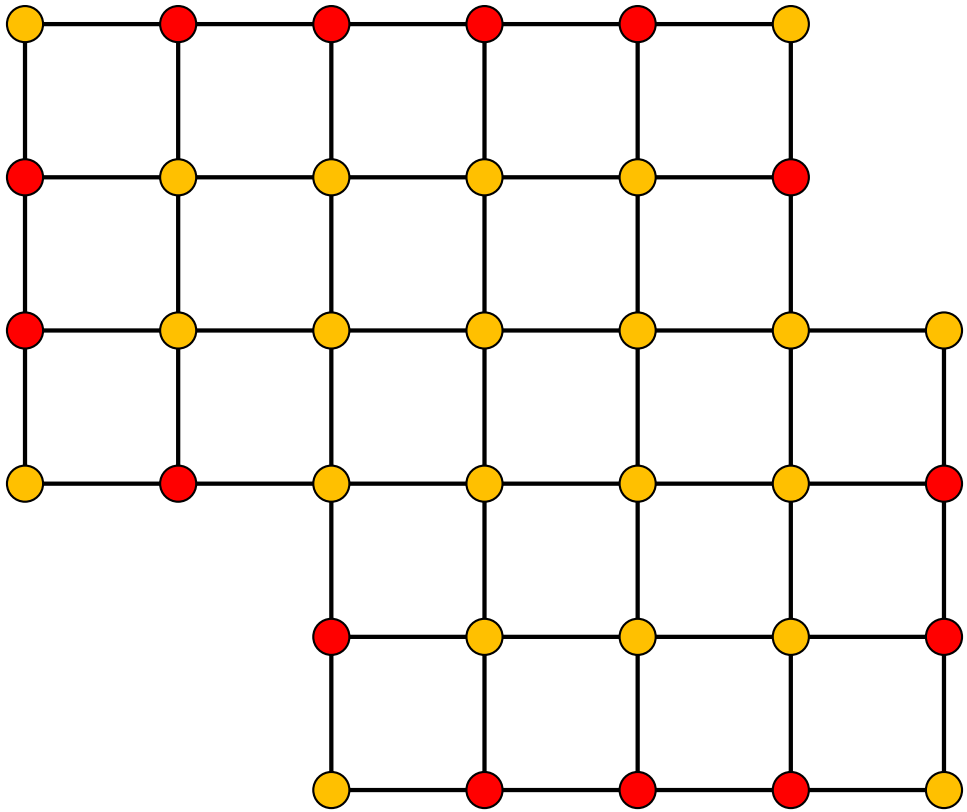
The neighborhood.

The Security Guard Problem



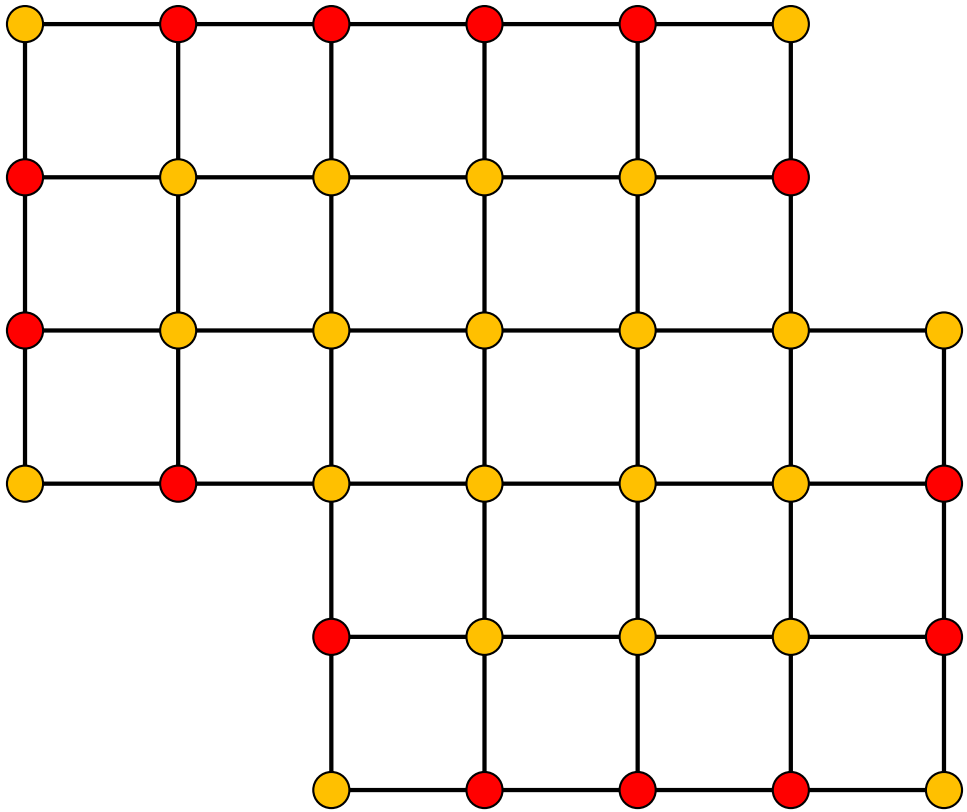
The neighborhood as a graph.

The Security Guard Problem



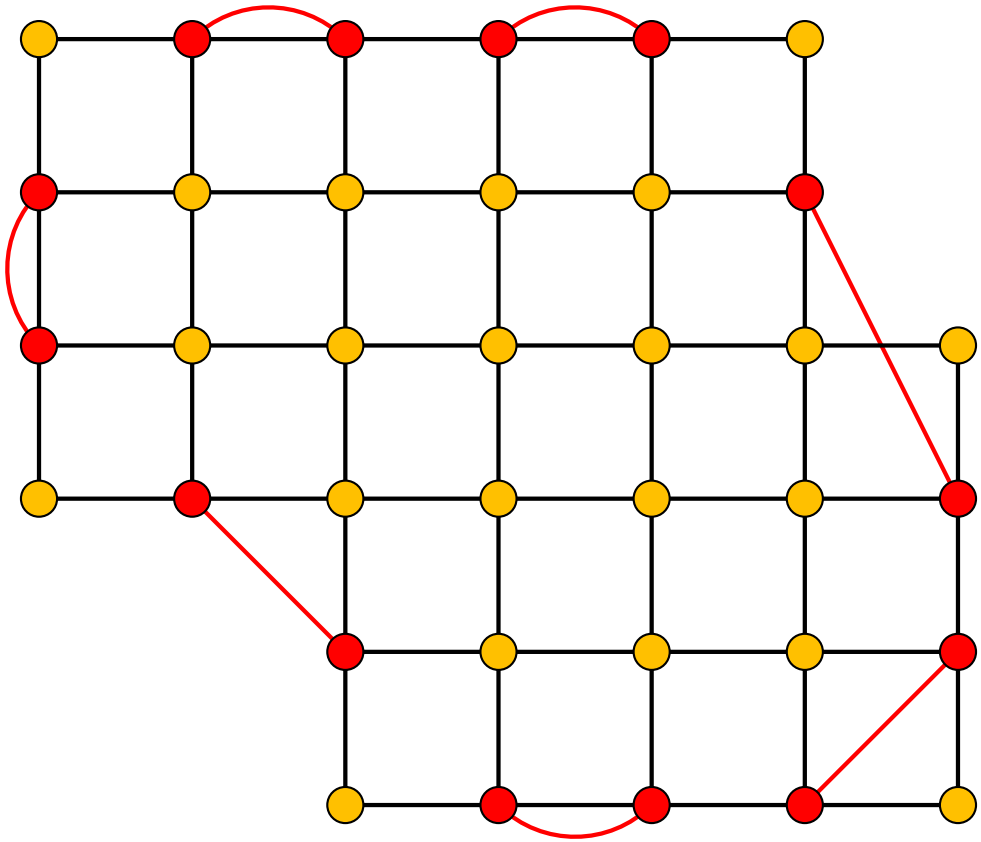
There are 14 odd vertices.

The Security Guard Problem



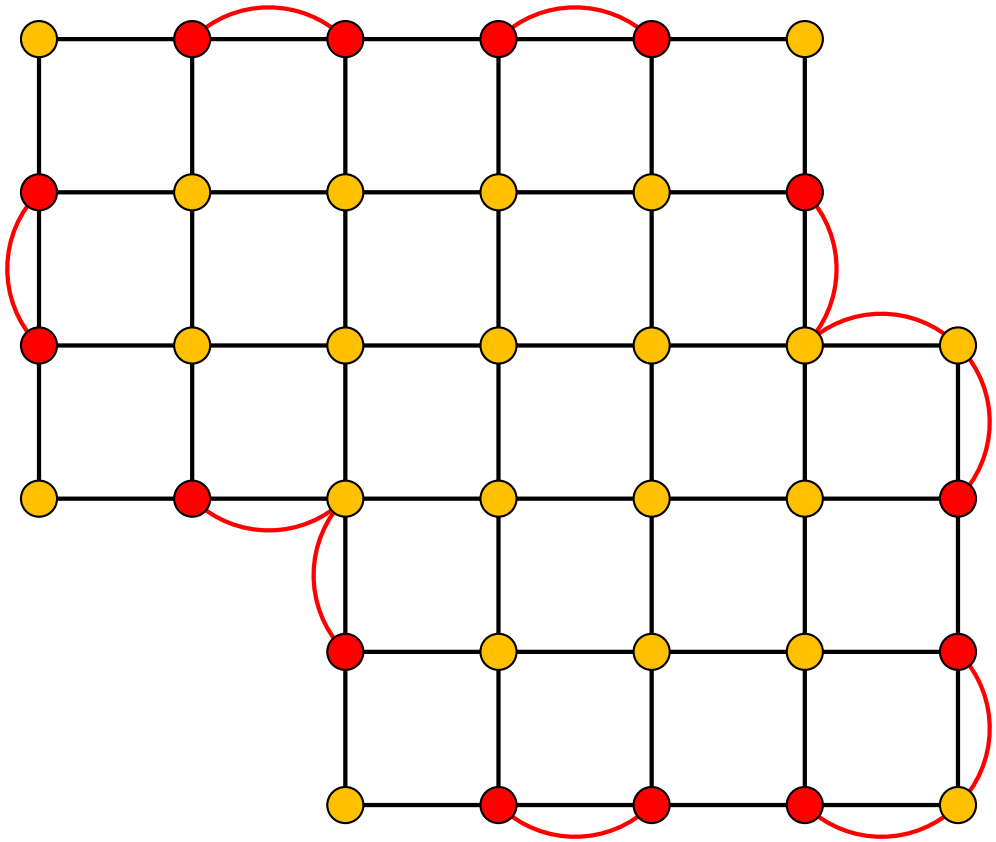
At least 7 edges must be added.

The Security Guard Problem



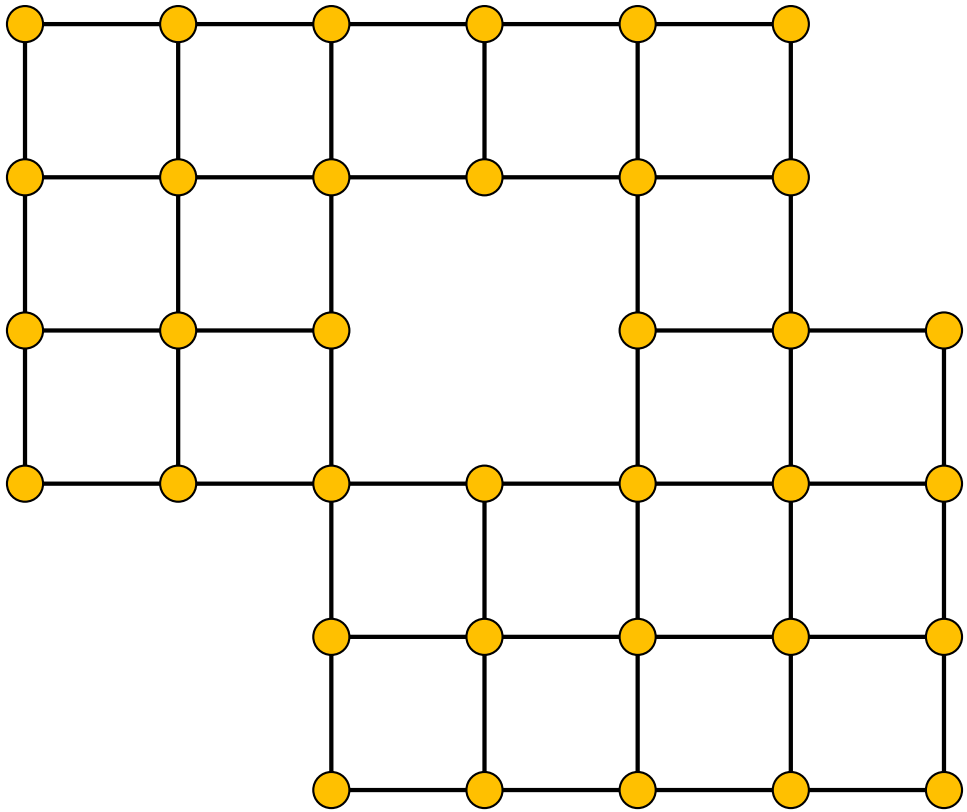
This “solution” is theoretically possible, but not practical. Why?

The Security Guard Problem



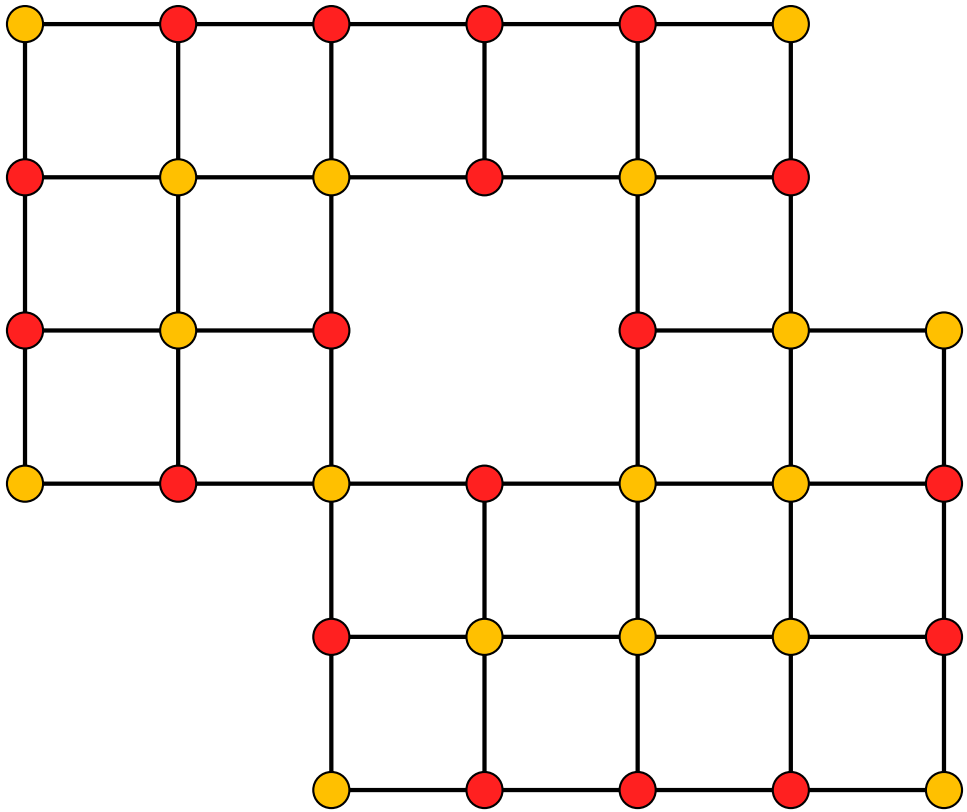
This solution uses 11 new edges. Is it optimal?

The Security Guard Problem



What if there were a city park?

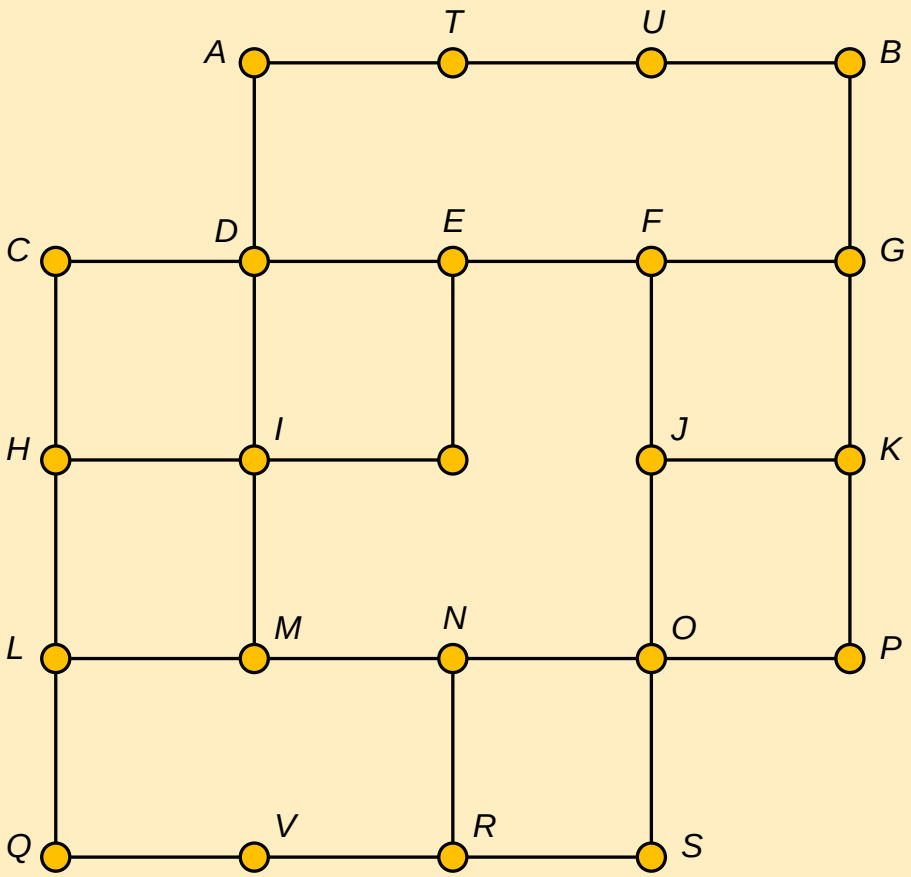
The Security Guard Problem



Now there are 18 odd vertices.

Example

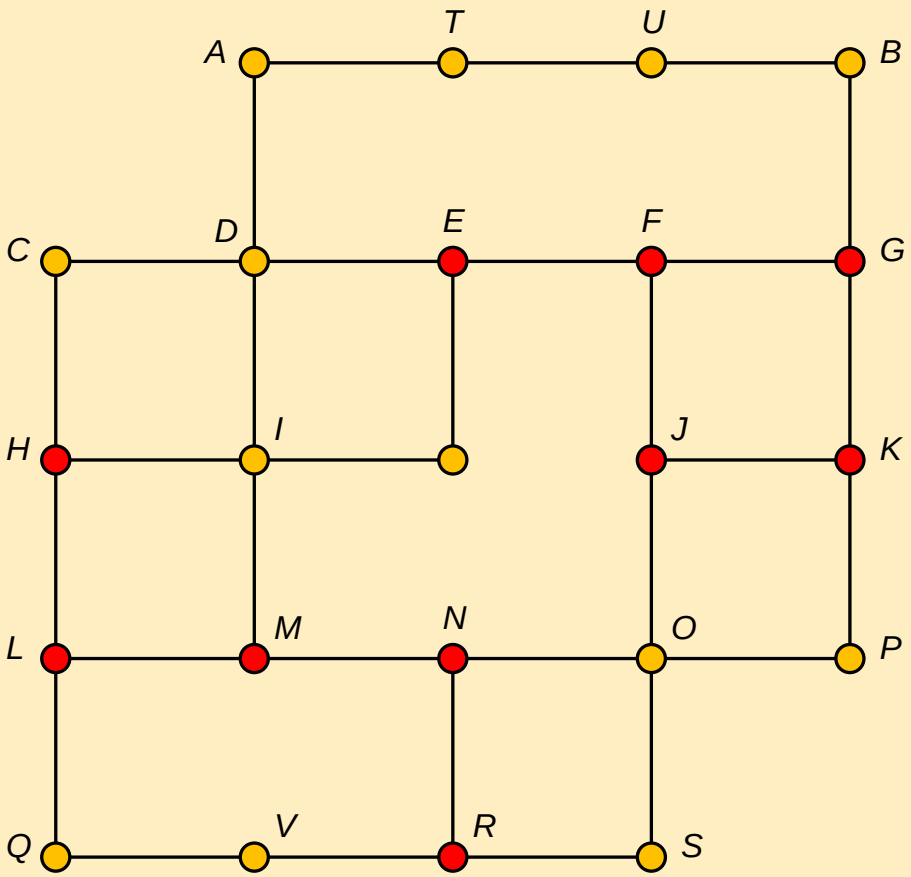
Eulerize This!



Eulerize this!

Example

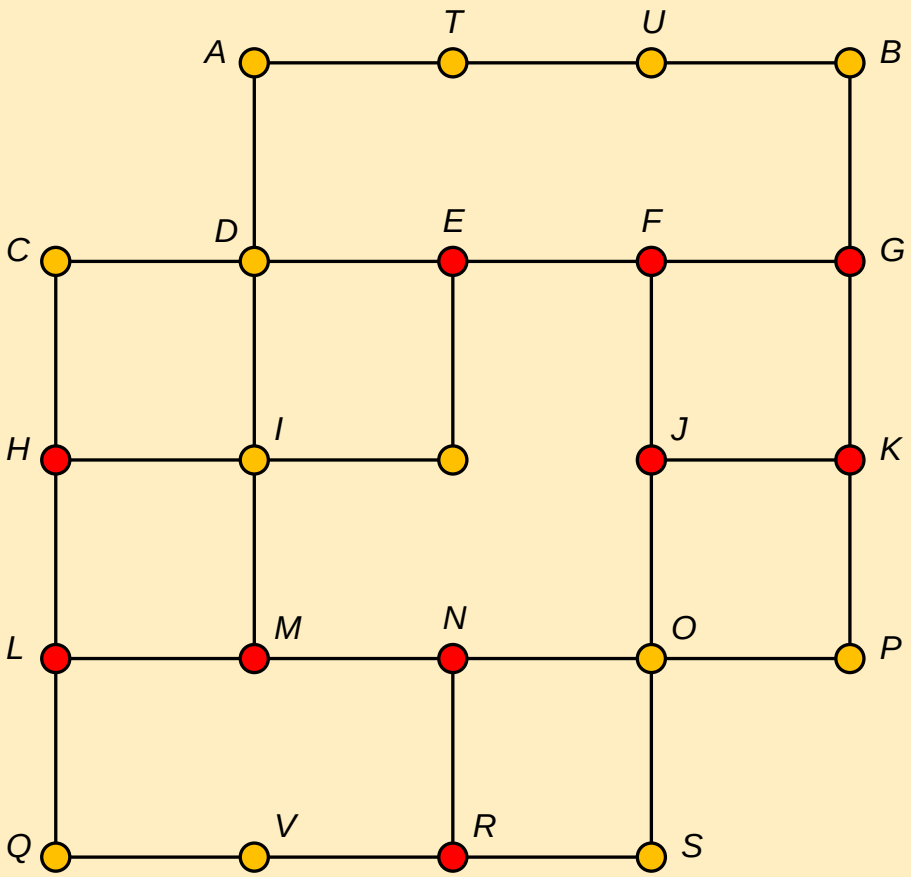
Eulerize This!



Eulerize this!

Example

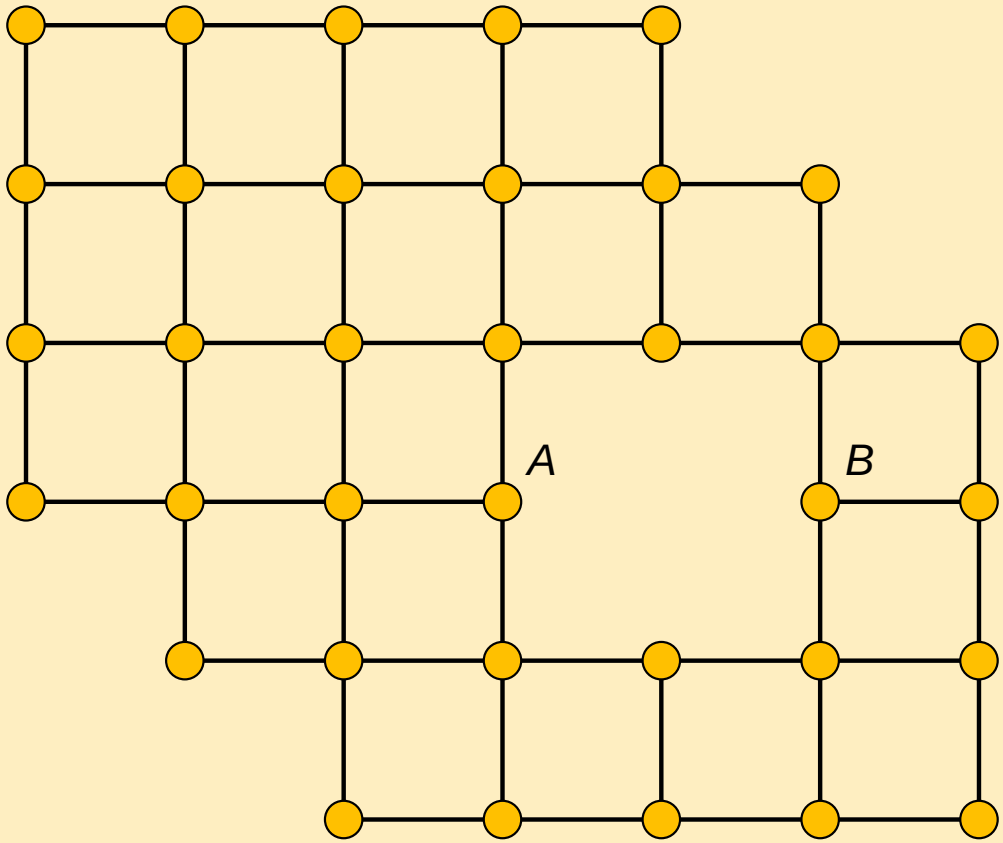
Eulerize This!



Now semi-eulerize it, starting at *M* and ending at *N*

Example

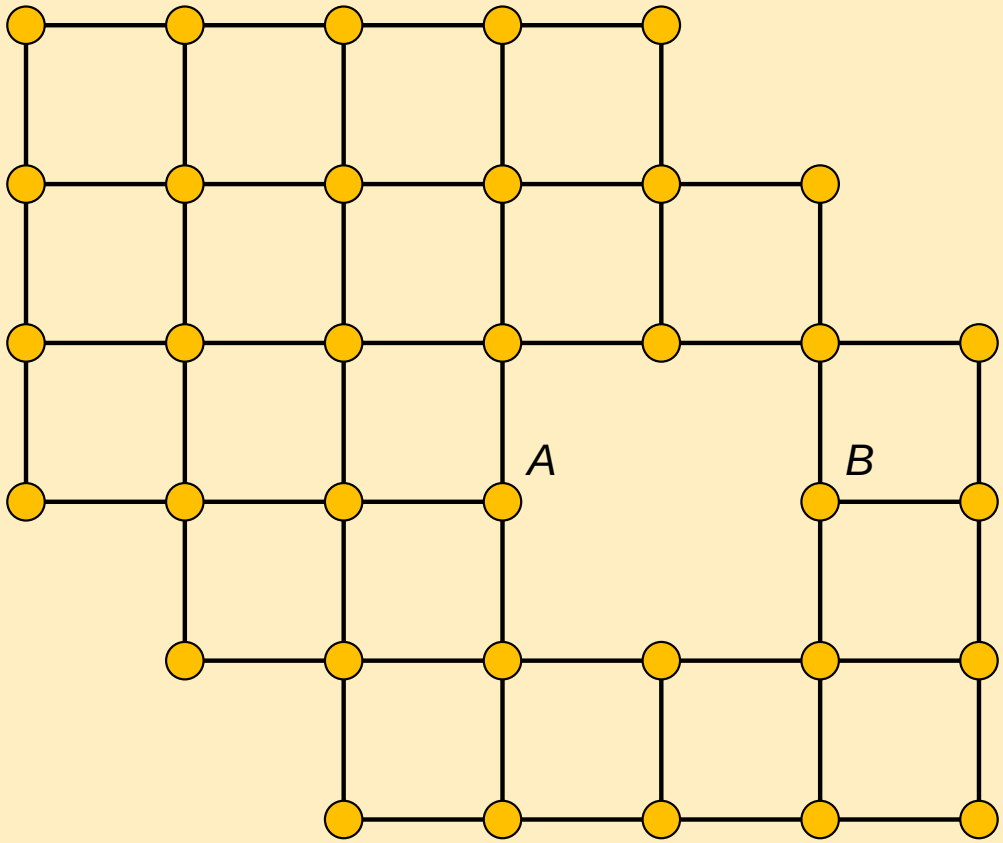
Eulerize This!



Eulerize this!

Example

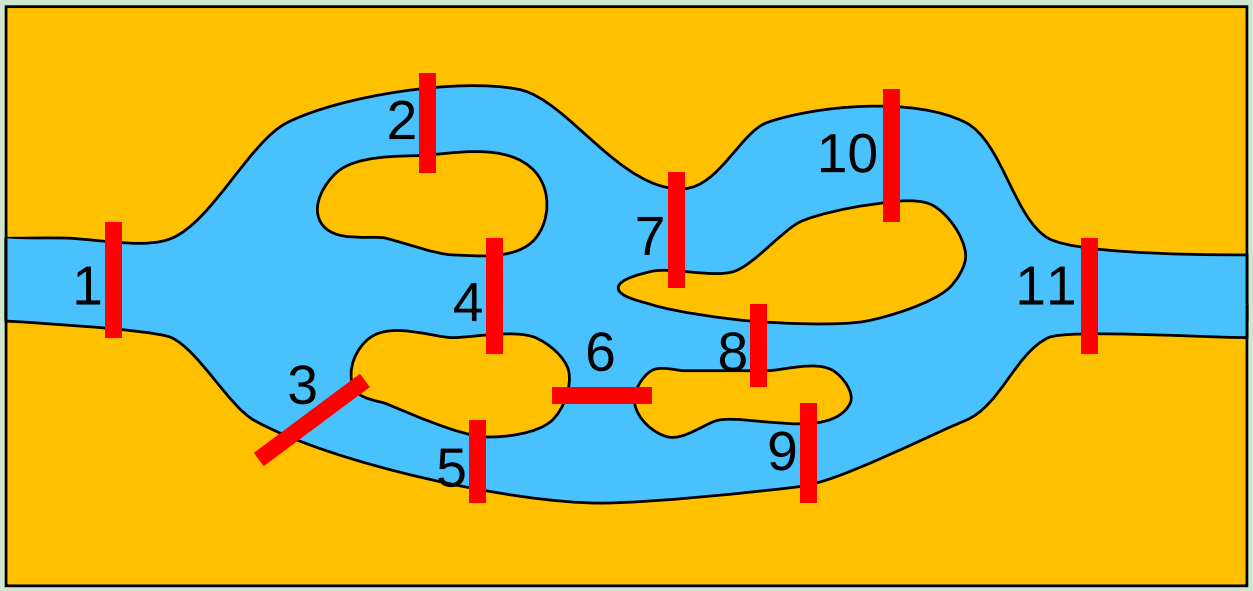
Eulerize This!



Now semi-eulerize it, starting at A and ending at B

The Bridges of Madison County Problem

Example (The Bridges of Madison County Problem)



- Eulerize the Bridges of Madison County.

Outline

Assignment

Assignment

- Chapter 5: Exercises 43, 44, 45, 47, 53, 54, 55.